Congestion Control and Channel Assignment in Multi-Radio Wireless Mesh Networks

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Abstract— **We address the problem of congestion control in multi-radio, multi-channel, wireless mesh networks. Compared to its single radio counterpart for which solutions exist, this problem is significantly more complex because it requires the radio channel assignments and the traffic allocations per channel be jointly optimized. We address the problem by introducing a formulation that allows its decomposition in two subproblems: A congestion control subproblem for traffic allocation to a fixed channel assignment over a node path and a discrete combinatorial channel assignment subproblem. We solve the conditional congestion control subproblem by mapping it to an optimization problem of traffic distribution to a set of radio paths. The solution provides channel congestion information that is utilized to address the channel assignment subproblem. This leads to an iterative procedure which guarantees successive increases to overall network utilization. Compared to existing work on multi-radio, multi-channel mesh networks, we show that our approach can yield significant gains both in terms of network utilization and establishing fairness.**

I. INTRODUCTION

Mesh networks are being deployed with multiple radios operating in orthogonal channels in order to achieve higher speeds.[1] In addition to the potential for interference mitigation via use of multiple channels, such architectures also introduce a yet unexplored flexibility to spatially allocate resources (radios and channels) to achieve fairness and congestion control objectives.

In this paper, we address the joint congestion control and channel assignment problem with an iterative, decomposition approach. Our *joint* optimization of rates and channel assignments incorporates that the latter is a discrete problem in nature, representing a significant departure from prior work. Our *iterative, decomposition* solution first determines a congestion-control driven channel assignment, and then for a given assignment, achieves the best distribution of traffic over the possible combinations of radios (i.e., logical paths). The iteration completes when we adapt channel assignments to ensure a higher total utility. In particular, by deriving feasibility conditions for the congestion control problem under a given channel assignment, we provide a significant "awareness" of the actual congestion limits of the multi-radio network, directly impacting the rate updates and the converged solution. Subsequently, by solving the channel assignment subproblem guided by the congestion control information of the previous problem solution, we derive guarantees that a new channel assignment yields an increase in the network fairness (utility) objectives. Our contributions are as follows.

[1]See for example, Mesh Dynamics and BelAir Networks.

First, for the congestion control subproblem, we account for the multi-radio multi-channel nature of the network by reducing the subproblem to one of distributing traffic to an appropriately selected set of radio paths. We construct this set in such a way that we also provide a solution to the traffic distribution problem at the different radios. When convergence takes place, the solution not only provides the transmission rates for each radio-to-radio link, but also determines which portion of the rate is designated for transmission in which radios (for all links along the multi-hop route), which can operate in different channels, under their own interference and congestion conditions.

Second, we solve the subproblem of channel assignment to radios by exploiting the congestion control information. We show that Lagrange multipliers, as an instance of the interaction of the two subproblems, can 1) *locate* the deficiencies of the previous iteration and motivate new changes 2) provide a local *classification* of the channels, a classification that apart from congestion, also reflects the impact of channel assignment on the global network fairness objective. We propose channel assignment algorithms that operate transparently to the notion of fairness, and provably guarantee successive increases to the network fairness objective. The results of the channel assignment subproblem in turn determine the new interference conditions and shape the congestion control problem of the next iteration.

We show that this interaction in solving the two subproblems is crucial to the performance of the joint solution. In comparison with existing work which addresses fairness issues in the multi-radio context, our approach can achieve significant gains of network utilization, while addressing a wide class of fairness objectives.

Related Work: Congestion control has been widely studied as a utility maximization problem in the context of wired networks, e.g., [15], [18], [19]. Studies for *wireless* networks using the utility maximization framework include congestion control design under asymmetries due to carrier sense [8], joint design of congestion control and power control [5], incorporation of clique-feasibility constraints [23], joint design of congestion control and MAC [4],[25], and joint design of congestion control and scheduling [6],[16]. While joint optimization problems have been previously addressed, our joint problem is unique in that it is of a discrete, combinatorial nature. Moreover, none address the aforementioned challenges that arise in multi-radio multi-channel networks.

Utility maximization models have also been employed for multi-path routing in wired networks, e.g., [12], [14], [17],

[24]. While multiple radios indeed provide multiple paths, [12], [14], [17], [24] do not incorporate the spatial resource allocation aspect of the problem that arises due to wireless channels.

Channel assignment in multi-radio networks has been studied with the objective of load-aware, interference-avoiding channel assignment [21],[22]. However, neither interference measures [21],[22], nor traffic load [22], are indicative of the network fairness objectives as considered here. Fairness objectives and channel assignment were taken into account in [2], together with throughput maximization and routing. However, in contrast to [2], we adopt a congestion-control oriented approach which results in improved incorporation of the congestion limits of the multi-radio multi-channel resources of the network. (We compare with [2] in Section V.) Finally, recent approaches employ exhaustive search for the channel assignment problem [20], yet are applicable only for networks of small size. To the best of our knowledge, we are the first to *propose* channel assignment algorithms jointly interacting with a multi-radio, multi-channel congestion controller.

The remainder of this paper is organized as follows: In Section II we describe our network model and formulate the problem. Section III introduces the radio path generation technique and addresses the congestion control sub-problem given a channel assignment. Section IV introduces a channel assignment algorithm that utilizes congestion control information. Section V provides simulation results and Section VI concludes.

Notation	Description
N	Number of Nodes
M_i	Number of node i's radios
E	Number of links between nodes
L	Number of links between radios
K	Number of channels
π	Channel Assignment to radios
S	Number of traffic sources
x_s	Transmission rate of source s
$R_{l,s,p}$	Binary variable denoting routing of traffic from path p of source s through link l
D	Maximum route size in the network
P_s	Radio paths for traffic distribution of source s
$x_{s,p}$	Transmission rate of source s at path p
C_j	Capacity of clique j
λ_j	Congestion price for clique j
σ_j	Available capacity at clique j
$F_{i,j}$	Binary variable indicating whether link i between two radios belongs in a clique j
Add/ Rmv	Set of links between radios that are created/broken due to a modification of the channel assignment
H_l	Set of links between the endpoints of l, operating in different channel from l

Fig. 1. Notation Table

II. Network Model

We consider a static wireless mesh network with a set of nodes denoted by $\mathcal{N}$, with $N = |\mathcal{N}|$. Each node $n \in \mathcal{N}$ is equipped with M_n identical radios. Ability for successful transmission between nodes within wireless range is denoted by a set of logical node-to-node links $\mathcal{E}$, with $E = |\mathcal{E}|$. The graph $(\mathcal{N}, \mathcal{E})$ is referred as the *network graph*.

Each link $e \in \mathcal{E}$ consists of one or more radio-to-radio logical links l, formed between the radios of e's endpoints. The set of all radio-to-radio logical links in the mesh network is denoted by $\mathcal{L}$, with $L = |\mathcal{L}|$. For each radio-to-radio link l, we assume that data is transmitted at a constant rate c_l. We assume stationary channel conditions and low mobility so that connectivity and transmission capabilities remain fixed.

The network operates with K orthogonal channels of equal bandwidth. An instance of a channel assignment to the radios of the network is denoted by: $\boldsymbol{\pi} = \{k_{n,i}, \quad i = 1, \ldots, M_n, \quad n = 1, \ldots, N\}$, where the radio i of node n is operating at channel $k_{n,i}$. We consider a slot-synchronized system with a periodic frame consisting of multiple slots. A node cannot transmit or receive on the same radio at the same slot and simultaneous operation of different radios of the same node at the same slot is permitted only if they operate at different channels. In addition, if the transmitter node of one link is within range of the receiver node of another link, then the links can transmit at the same slot only on radios of these nodes that have been assigned to different channels. Finally, incoming traffic to a radio of a node can be immediately forwarded for transmission to a different radio (and channel) of the same node.

We consider a set of sources $\mathcal{S}$, with $S = |\mathcal{S}|$, originating from network nodes and share the mesh network. The utility of a source transmitting at average rate x_s is expressed by a well-known family of utility functions:

$$U(x_s) = \begin{cases} w_s \frac{x_s^{1-\alpha}}{1-\alpha}, & \text{if } \alpha \neq 1 \\ w_s \log x_s, & \text{otherwise} \end{cases}$$

where $U(\cdot)$ is a strictly concave, non-decreasing, twice differentiable function. Finding a source rate vector that maximizes aggregate utility can lead to realization of various fairness objectives. The fairness region depends on the priority parameters $\mathbf{w} = \{w_s, \quad s = 1, \ldots, S\}$ and parameter α. For example, $\alpha = 0$ leads to throughput maximization, $\alpha = 1$ to proportional fairness, $\alpha = 2$ harmonic mean fairness and $\alpha = \infty$, max-min fairness.

Each source s is associated with an origin-destination node pair denoted by (h_s, d_s). We consider fixed node-to-node routing, expressed by binary variables $R_{l,s}$ that are equal to one if the route of source s is using link l and zero otherwise. Although node-to-node routes are fixed, it is possible to split traffic across radios and channels of each path. We denote by D the maximum route size in the network. In the case of mesh networks where traffic is routed to wired gateways, D does not typically exceed 6 hops.

Given the node-to-node routing, we exclude from the set $\mathcal{E}$

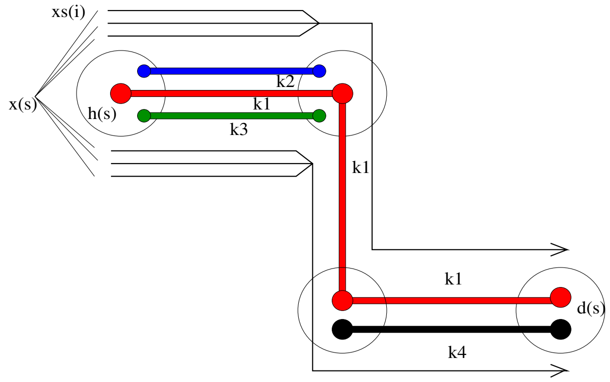


Fig. 2. Construction of the set of paths

those node-to-node links that are not included in the route of any source and their corresponding radio-links from $\mathcal{L}$. We also denote by Π, the space of all acceptable channel assignments, in the sense that they establish the necessary connectivity, i.e. at least one common channel exists between two nodes $(n, n') \in \mathcal{E}$.

Problem statement: We first discuss the multi-radio congestion control problem by casting it in an abstract manner. We will provide rigorous formulation when proceeding to its solution in the next sections. Let P denote the traffic distribution options in the network for a given channel assignment and σ_j the available capacity of resource j (capacity minus service demand). The joint congestion control and channel assignment problem can be described as follows:

MRMC-CC-CA:

$$\max_{\mathbf{x_s}(\mathbf{P}), \boldsymbol{\pi} \in \Pi,} \quad \sum_{s=1}^{S} U(x_s(P(\boldsymbol{\pi})))$$

$$s.t. \quad \sigma_j(\boldsymbol{\pi}, \mathbf{x}, P(\boldsymbol{\pi})) \geq 0, \quad \forall j$$

The difficulty in solving problem MRMC-CC-CA stems from the discrete combinatorial nature of the channel assignment decisions which can be formulated as integer variables and determine the channel participation of radios and links. Simpler combinatorial problems for multi-radio networks such as channel assignment that realizes a given set of rate demands have been shown to be NP-hard [22] while in our case the optimal source rates of the globally solution are not known. We adopt a decomposition approach which divides the problem in a congestion control subproblem and a channel assignment subproblem. The congestion control subproblem is subject to a fixed channel assignment and accounts for conditions that help in distributing traffic up to the congestion limits of the actual multi-radio, multi-channel, network resources. Based on the computed source rates, the channel assignment subproblem utilizes information such as the Lagrange multipliers and link utilization from the congestion control subproblem solution to re-optimize the radio channel assignments in a beneficial direction for the network. The two subproblems are sequentially and iteratively solved until termination.

III. Congestion Control

In this section we address the congestion control subproblem subject to a fixed channel assignment $\boldsymbol{\pi}$. The main goal for this non-linear programming problem is to derive rate adaptation updates whose converged solution accounts and exploits transmissions from multiple radios, traffic distribution options, and channel-dependent interference. This is achieved by constructing a set of radio paths for traffic distribution and by deriving an appropriate set of feasibility conditions for traffic to share channels and time slots.

Path Construction: Multiple radios empower channel assignment decisions to spatially distribute the network capacity. While the advantage of exploiting multiple transmission options through different radios and channels is obvious, the exact way traffic should be distributed in the most efficient way is not clear. The *total* incoming traffic at each intermediate node is arriving from multiple radios and should be split to each of the outgoing links which might also be incident to different radios. In addition, the *per source* distribution of incoming/outgoing traffic for each radio has to be determined in conjunction with the resultant aggregate source rate and its fairness requirements while also accounting for the impact of the induced interference on other flows within transmission range.

We construct a set of radio-to-radio paths for each node-to-node route s as follows. Starting from the link of the route e that is adjacent to the source s, we create one path p for each common channel between its two end nodes $tr(e), rcv(e)$. For subsequent links of the route e', for each common channel between its end nodes, we append the corresponding radio-to-radio link to all the paths constructed for the previous link. This incremental path construction procedure iterates for all links until the end of the node-to-node route and results in a set of paths $\mathcal{P}_s$ for each source s (Fig. 2).

Our congestion control design distributes the traffic of each source s across its set of paths $\mathcal{P}_s$. Based on the construction procedure, the number of paths $P_s = |\mathcal{P}_s|$ of each route s is equal to the product of common channels at each link along the route. This number is loosely upper-bounded by $(\max_{i \in N} M_i)^D$ where D is the maximum route length in the network. For example, in Figure 2 route s has $P_s = 3 \times 1 \times 2 = 6$ radio paths as opposed to the upper bound of 27. One consideration would be that a large P_s would cause a very slowly converging congestion control algorithm to an extend of not being implementable. For example, in a 10-hop route of 2-radio nodes, P_s is in the order of 1024 paths. Here, the mesh architecture hypothesis is crucial. Every node is within D hops of a gateway and in typical deployments D is rarely greater than 5-6 hops. Hence, we consider that for multi-hop networks of mesh type, such an approach can be viable.

A source s perceives utility $U(x_s)$ when data are transmitted from $h(s)$ to $d(s)$ at a total rate of x_s. Rate x_s is the aggregate traffic achieved by transmission to each radio path $p \in P_s$ with rate $x_{s,p}$, hence $x_s = \sum_{p=1}^{P_s} x_{s,p}$.

We denote by $\mathbf{x_s} = \{x_{s,p}, \quad p = 1, \ldots, P_s\}$ the source-distribution vector, and by $\mathbf{X} = \{\mathbf{x_s}, \quad s = 1, \ldots, S\}$ the network-distribution vector. In addition we use the binary routing variables $R_{l,s,p}$ indicate if radio-to-radio link l is used by the path p of source s or not.

The rate of a radio-to-radio link equals the sum of the individual rates of all the paths of the network crossing this link. Those individual rates also indicate the portion of the aggregate traffic of the link that should be routed in each radio path.

Feasibility Conditions: We use a generic contention graph CG_0 to describe interference relationships in the network graph. Each vertex in CG_0 corresponds to a node-to-node link in the network graph and each edge corresponds to two potentially interfering links in the network graph (transmitter node of one link is within range of the receiver node of the other link) assuming there is only a single channel in the network.

Let N_{cl}^0 be the total number of maximal cliques in CG_0. Since it is possible for all links of each maximal clique to be used in all channels, we replicate each maximal clique K times, resulting in a total of $N_{cl} = K * N_{cl}^0$ maximal cliques, viewed as potential resources shared by the radio-to-radio links. Since each clique maps to a single channel, the feasibility conditions for any network distribution vector $\mathbf{X}$ should impose the normalized aggregate load on each clique Φ_j not to exceed a normalized capacity C_j:

$$\sum_{s=1}^{S}\sum_{p=1}^{P_s}\sum_{l=1}^{L} R_{l,s,p}F_{l,j}\frac{x_{s,p}}{c_l} \leq C_j, \qquad \forall j = 1,\ldots,N_{cl} \quad (1)$$

where the binary variables $F_{l,j}$ depend on the radio channel assignments and indicate whether radio link $l = 1,\ldots,L$ belongs to clique j, $j = 1\ldots,N_{cl}$.

The clique capacities C_j should guarantee the existence of a time slot schedule realizing the radio link loads induced by $\mathbf{X}$. Our clique formulation implies that cliques in each channel will be scheduled independently in the time domain. This allows leveraging results of single channel systems to determine capacities that ensure schedulability. More specifically, it has been shown in [10] that setting $C_j = 0.46$ for each clique ensures such sufficient feasibility conditions. On the other hand, if CG_0 is a perfect graph, then a maximum utilization factor $C_j = 1$ yields both sufficient and necessary conditions (i.e. the constraints can capture *all* feasible allocations $\mathbf{X}$)[13].

Our clique-based formulation introduces the complexity of computing all maximal cliques in a graph which in general is an NP-complete problem and a time-consuming computation in practice. However, this is a one-time computation, which we deem reasonable for the static mesh network setting. In Section V, we enumerate all maximal cliques for fairly large networks that may arise in practice. Alternatively, we could utilize a set of link-based constraints where the aggregate normalized traffic of each link and all its interfering links in each channel is less than unity [2]. This set of constraints does not require the clique computation step. However, since not all interfering links of link l are within range of each other this set of costraints can be overly conservative. We show in Section V that it can lead to severe network under-utilization.

Given radio paths and feasibility conditions we formulate the congestion control subproblem as the following utility-maximization problem:

MRMC-CC:

$$\max \sum_{s=1}^{S} U(\sum_{p=1}^{P_s} x_{s,p})$$

$$\text{s.t.} \quad \sum_{s=1}^{S}\sum_{p=1}^{P_s}\sum_{l=1}^{L} R_{l,s,p}F_{l,j}\frac{x_{s,p}}{c_l} \leq C_j, \quad j = 1,\ldots,N_{cl}$$

Each source s is physically located at a given node and perceives satisfaction from individual transmissions over the multiple combinations of traffic distribution in its path set $\mathcal{P}_s$. Hence $U(\cdot)$ is a strictly concave function of x_s but not the variables $x_{s,p}$. From a mathematical standpoint, the lack of strict concavity with respect to the variables $x_{s,p}$ prohibits optimization solutions used in previous single-channel clique-based models (e.g., [7],[4],[8]). Even when congestion prices converge the source updates would cause oscillations and the original problem will never be solved.

Our radio-path based formulation enables convergence to a unique solution with a technique previously used to address multi-path routing problems in wireline networks [17],[24],[15]. The objective function is modified, with small penalty quadratic terms: $-\delta \sum_{s=1}^{S}\sum_{i=1}^{P_s}(x_{s,p})^2$, where δ is a small positive constant. These terms cause a small deviation from the optimal solution of *MRMC-CC*, however the objective function becomes strictly concave with respect to each $x_{s,p}$. However, only smooth convergence within a certain deviation from the optimal solution of *MRMC-CC* is guaranteed so far. *Exact* solutions to the original problem *MRMC-CC* can be achieved, with the use of Proximal Optimization Theory [3], similarly with existing approaches for wired networks [17]. According to these approaches, additional outer loops are iteratively used for eliminating the deviation effect that is due to the quadratic terms. More precisely, an additional variable $z_{s,p}$ is associated with each $x_{s,p}$ and the trasformed problem is as follows:

MRMC-CC exact:

$$\max \sum_{s=1}^{S} U(\sum_{p=1}^{P_s} x_{s,p}) - \delta \sum_{s=1}^{S}\sum_{p=1}^{P_s}(x_{s,p} - z_{s,p})^2$$

$$\sum_{s=1}^{S}\sum_{p=1}^{P_s}\sum_{l=1}^{L} R_{l,s,p}F_{l,j}\frac{x_{s,p}}{c_l} \leq C_j, \qquad \forall j = 1,\ldots,N_{cl} \quad (2)$$

The solution to *MRMC-CC exact* can be found following analogous steps as in [17] by considering the Lagrange multipliers (congestion prices) $\boldsymbol{\lambda}$ associated with each constraint. The procedure can be summarized as follows (see [17] for detailed description).

At each iteration k,

1) The distribution of the traffic of source s, to the radios of the path p is given by:

$$x_{s,p}(k+1) = \arg\max \Bigg\{ U(\sum_{p=1}^{P_s} x_{s,p}) - \sum_{j=1}^{N_{cl}} \{\lambda_j \times$$

$$R_{l,s,p} F_{l,j} \frac{x_{s,p}}{c_l}\} - \delta \sum_{i=1}^{P_s} (x_{s,p} - z_{s,p})^2 \Bigg\}, \quad (3)$$

$$x_s(k+1) = \sum_{p=1}^{P_s} x_{s,p} \quad (4)$$

2) For $j = 1, \ldots, N_{cl}$ the Lagrange multipliers are updated according to :

$$\lambda_j(k+1) = \Bigg[\lambda_j(k) + \quad (5)$$

$$\gamma \left(\sum_{s=1}^{S} \sum_{p=1}^{P_s} \sum_{l=1}^{L} R_{l,s,i} F_{l,j} \frac{x_{s,p}}{c_l} - C_j \right) \Bigg]^+$$

where γ is a sufficiently small step size. After convergence, the variables $z_{s,p}$ take the converged value $\bar{x}_{s,p}$ of the corresponding variables $x_{s,p}$ and the entire process above is repeated until convergence.

Problems related to multi-path routing have been addressed before in the wired networks literature. Our contribution does not lie in showing how to solve multi-path routing problems but in formulating and reducing the multi-radio congestion control subproblem with the path construction technique.

As the congestion controller distributes traffic in a clique, it converges to some rates for each of its radio-to-radio links. Those rates, being the sum of multiple individual paths that express different radio transmission combinations, also indicate the portion of the traffic that should be distributed to each of the paths. All along the multi-hop route, traffic of that link will be carried to radios that operate in under distinct interference and coexistence with other flows. However, we want to highlight that the congestion controller will converge to such a solution in the rate distribution problem to the different radios and channels, that will be 'optimal' in the sense of better meeting the fairness objectives, as expressed by the network aggregate utility.

IV. Channel Assignment

The congestion control sub-problem yields optimal radio paths and source rates given a channel assignment. Except for very small networks, reaching the optimal solution of the joint problem through exhaustive search is not feasible due to the large number of channel assignments (order of $\sum_{n=1}^{N} (M_n)^K$)).

We propose an approach where congestion control and channel assignment sub-problems are solved sequentially and iteratively. Given a solution $\mathbf{X}$ of the congestion control sub-problem that results from channel assignment $\boldsymbol{\pi}$, we seek a new channel assignment $\boldsymbol{\pi'}$ that will yield a congestion sub-problem solution $\mathbf{X}'$ of higher aggregate network utility in the next iteration.

In principle, the new channel assignment $\boldsymbol{\pi'}$ should remove traffic from highly congested resources or add bandwidth to highly congested resources if possible. In a multi-radio mesh network these two actions translate to channel modifications that result in deletion of radio links from highly congested channels and cliques or addition of radio links on other channels, respectively. An intuitive attempt to realize this high-level goal would be to formulate and run a global channel allocation optimization problem that minimizes network-wide interference subject to traffic vector $\mathbf{X}$. However, this approach would not necessarily yield a channel assignment that results in increase of the network utility function.

We propose a heuristic for the channel assignment sub-problem which uses congestion control information and guarantees successive increases in network utility. The key idea of our channel assignment algorithm is to use the Lagrange multipliers of the congestion control sub-problem to identify the most congested cliques as local areas of highest priority. This approach focuses the algorithm search at a level local to a clique. Within these cliques local channel modifications are sought that result in links deleted from the congested clique or by radio links added on other channels for reinforcement.

From the optimization problem viewpoint, the channel modifications result in modifications of the discrete constraint coefficients F_{lj} thus producing a new set of constraints for the congestion control sub-problem of the next iteration. A channel modification of even a single radio link not only modifies the constraints of the cliques it belongs but also the constraints of other cliques in different channels. The challenge is to find the channel modifications that will guarantee an increase in the network utility function without solving the congestion control sub-problem for each potential modification. Our algorithm identifies such channel modifications by utilizing the traffic vector $\mathbf{X}$ of the congestion control sub-problem.

A. Local channel modifications

We first identify the minimal channel modifications that result in radio link deletions or additions. We then identify a set of conditions that need to be satisfied in order for such channel modifications to yield higher aggregate network utility of the congestion control sub-problem of the next iteration. Finally we introduce a channel assignment algorithm that incorporates the channel modifications and conditions.

Consider a congested clique j that operates in channel k_j. We seek minimal local channel modifications that either delete radio links from clique j and channel k_j or reinforce clique j by adding radio links to other channels that share common *node* endpoints with the links of clique j. The minimal channel modifications can be link-based or radio-based. Link-based modifications involve switching both radios of a radio link to a different channel. Radio-based modifications involve switching only a single radio to a different channel. Link-based modifications are more drastic because they result in more links switching channels. For ease of illustration in the following we describe radio-based channel modifications for link deletion and link reinforcement at clique j. Link-based channel modifications are performed in a similar manner.

Link deletion: A radio of link l in clique j switches from channel $k_l(=k_j)$ to channel k'. The new channel k' should be different than the channels assigned to the other radio links of node-to-node link e_l where radio link l belongs. This modification results in deletion of radio link l and all adjacent links of its switched radio on channel k_l. At the same time this modification may result in addition of new links adjacent to the radio if there exist other radios within transmission range in channel k'.

Link reinforcement: Let $e_{l'}$ be the node-to-node link where a radio link l' of clique j belongs. A radio of a "parallel" radio link l that also belongs to $e_{l'}$ switches from its channel k_l to channel k'. This modification results in deletion of links adjacent to this radio in channel k_l and addition of links adjacent to this radio in channel k'. The new channel k' should be different than the channels assigned to the other radio links of $e_{l'}$. Also, to ensure reinforcement, channel k' should be such that more "parallel" radio links to the links of clique j are added for channel k' than deleted from channel k_l.

B. Eligibility conditions

We now derive the conditions under which the above modifications result in a new channel allocation $\boldsymbol{\pi}'$ that results in an increase of the aggregate network utility function, given channel assignment $\boldsymbol{\pi}$ and the traffic distribution $\mathbf{X}$ of the congestion control sub-problem.

Let x_l be the load of each radio link l, and σ_j be the available bandwidth of each clique j:

$$x_l = \sum_{s=1}^{S} \sum_{p=1}^{P_s} R_{lsp} \frac{x_{sp}}{c_l}, \qquad l = 1, \ldots, L \tag{6}$$

$$\sigma_j = C_j - \sum_{l=1}^{L} F_{lj} x_l, \qquad j = 1, \ldots, N_{cl} \tag{7}$$

Consider a channel modification (either link deletion or link reinforcement). Let Rmv be the set of deleted radio links from channel k_l and Add be the set of new radio links in channel k'. For each deleted link $l \in Rmv$, denote H_l the set of radio links that have common node endpoints with link l and operate on different channels than k_l according to channel assignment $\boldsymbol{\pi}$. Also denote by Add_l a link in Add that has common node endpoints with link l in Rmv set and operates in channel k'. For each link l in Rmv, assume that we (independently) load each radio link l' in sets H_l and A_l with the load of link l, i.e. $x_{l'} = x_l$. The following conditions guarantee that the load can be supported by all channels (other than k_l) and all their cliques after the local channel modification from k_l to k':

$$\sum_{l' \in \Phi_j} x_{l'} \leq \sigma_j, \qquad j = 1, ..., N_{cl} \tag{8}$$

where $\Phi_j = \bigcup_{l \in Rmv} (H_l \bigcup Add_l) \bigcap j$ is the set of all added or existing radio links that shared common node endpoints with deleted links that belong to clique j.

Theorem 1: Let $\mathcal{U}(\mathbf{X}, \boldsymbol{\pi})$ denote the aggregate utility of the solution $\mathbf{X}$ under channel assignment $\boldsymbol{\pi}$. For every minimal channel modification $\boldsymbol{\pi} \rightarrow \boldsymbol{\pi}'$ that obeys conditions (8), the solution $\mathbf{X}'$ of the new congestion control sub-problem will yield $\mathcal{U}(\mathbf{X}', \boldsymbol{\pi}') \geq \mathcal{U}(\mathbf{X}, \boldsymbol{\pi})$.

Proof: Modifying a channel assignment signifies displacement of one or more radio links to different constraints of the original congestion control sub-problem, as well as deletion or addition of radio links in the constraints. If the local channel modification satisfies the conditions (8), all constraints in the new congestion control sub-problem under π' will be able to carry the traffic carried under π. This is due to the fact that the rate of all deleted links in set Rmv can be certainly served by other links without causing a decrease to the rate of any other flow in the network. Hence, it follows that the new aggregate converged rates for each of the radio-to-radio links will either be increased - if the congestion controller of the next iteration decides that this is of benefit - or in the worst case remain unchanged. The same argument holds for the converged rates of the paths and the sources as well, and by the increasing property of the utility functions, the result follows. ■

Example. Fig. 3 provides an example for the derivation of eligibility conditions (8) for a local channel modification where the radio of node B tuned to channel 1, switches to channel 2. This results in radio links BF(1),AB(1),BE(1) to be removed from channel 1 (Rmv set) and radio link BF(2) to be created on channel 2 (Add set). To derive the conditions we focus on channels 2 and 3. In the new channel assignment,

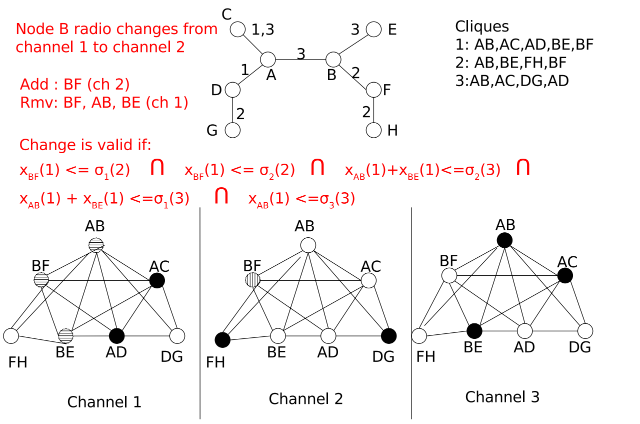


Fig. 3. Example: Derivation of eligibility conditions. There are three channels, each with three cliques. Black/white vertices: Links l of clique j with $F_{l,j} = 1/0$. Horizontal-striped vertices: Rmv set. Vertical-striped vertices: Add set.

the load of each radio link in the Rmv set should be carried by its corresponding existing links (the H_l sets) and/or added links (Add_l sets) in the other channels (channels 2 and 3). The eligibility conditions are derived by (i) "loading" the links from all sets $H_l \bigcup Add_l$ with the loads of Rmv set and (ii) finding the intersecting links of sets $H_l \bigcup Add_l$ and each of the six cliques in channels 2 and 3.

C. Channel assignment algorithm

We describe the algorithm that selects the channel reassignment at each iteration of the congestion control/channel assignment loop. Its input is the solution $\mathbf{X}$ (or link loads x_l) and clique Lagrange multipliers of the congestion control subproblem and the current channel assignment $\boldsymbol{\pi}$. Its output is a new channel assignment $\boldsymbol{\pi'}$ that aims at higher aggregate network utility in the next iteration.

The algorithm visits all cliques j in descending order of their Lagrange multipliers λ_j. For each clique j, the algorithm searches over all local channel modifications in terms of link deletion or link reinforcement and identifies a set I_j of eligible modifications using conditions (8). If no eligible modification is found the algorithm proceeds to the next clique. If multiple eligible modifications are found, the algorithm terminates by selecting the modification (and new channel assignment) that yields maximum movement of aggregate traffic: $\sum_{l \in Rmv} x_l$ for link deletions or $\sum_{l \in Add} x_l$ for a link reinforcement; the output channel assignment is applied and then input to the congestion control sub-problem of the next iteration. If no eligible channel modification is found for any clique, the algorithm terminates with a NULL output; this also signals termination of the congestion control/channel assignment loop since no further improvements can be guaranteed by channel assignment.

Complexity analysis. We provide a worst-case analysis of the channel assignment algorithm in terms of the maximum node (and radio) degree Δ in the network graph and the number of cliques N_{cl}, maximum clique size ϕ_l. The analysis also uses the fact that the maximum number of radios ϕ_n that have a radio link in a given clique equals $2\phi_l$.

Sorting the cliques in descending order of congestion price is of $O(N_{cl} \log(N_{cl}))$ complexity. While visiting a clique, at most ϕ_n radios will be examined. For each radio, the algorithm examines switching the channel of the clique in one of the remaining $K-1$ channels. The size of the sets Add and Rmv are bounded above by Δ. Thus each clique visit yields complexity of $O(K\Delta\phi_n)$. Since $\phi_n < 2\phi_l$ both link deletions and link reinforcements for each clique will be of complexity $O(K\Delta\phi_l)$. In the worst case the algorithm will visit all of the N_{cl} cliques. Taking into account the initial sorting complexity, the complexity of the channel assignment algorithm is $O(N_{cl}K\Delta\phi_l + N_{cl}\log N_{cl})$.

V. Numerical Results

We evaluate the performance of our approach and compare it to the approach of Alicherry et. al in [2], which introduced an optimization framework to jointly determine routes and channel assignments that optimize aggregate throughput of multi-hop flows subject to fairness constraints. We first describe the implementation of our iterative congestion control and channel assignment algorithms. Then we discuss the results of the comparative performance evaluation.

A. Implementation

We have implemented our joint congestion control and channel assignment framework using a custom simulator in C. We have also implemented the approach in [2] using CPLEX [1]. The implementation of our approach includes the following components.

Maximal clique enumeration algorithm. This algorithm is executed once at network initialization and determines the maximal cliques given node locations and interference relationships. Enumeration of maximal cliques is an NP-complete problem for arbitrary graphs but there exist efficient algorithms for graphs induced by wireless networks. We have implemented a greedy heuristic based on the approximate technique in [11]. This heuristic typically enumerates cliques for fairly large topologies (e.g. Fig. 5(a)) in less than a minute.

Initial channel assignment algorithm. The iterative procedure begins using an initial channel assignment that is not aware of the congestion control information. In principle, any interference-aware greedy heuristic for link channel assignment such as [21], [22] can be used for initialization. We have implemented a greedy heuristic that assigns channels to radios instead of node-to-node links. In contrast to previous approaches this allows traffic to be transferred simultaneously over different pairs of radios on the same link. Details can be found in [9].

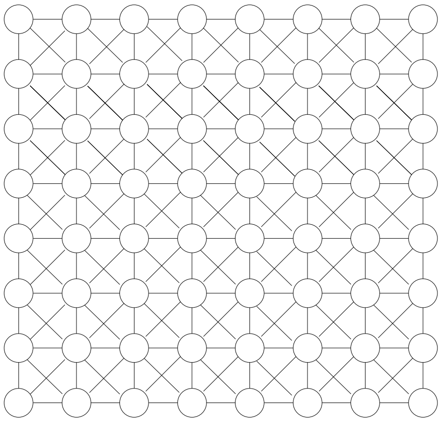
Congestion control algorithm. The congestion control algorithm operates subject to a fixed channel assignment and a set of computed radio paths. In contrast to using a solver like CPLEX our implementation is iterative and requires only simple algebraic operations, hence of lower complexity. This is enabled by the multi-path congestion control problem formulation presented in Section III.

Channel assignment algorithm. The channel assignment algorithm described in section IV is also of low complexity since it only involves local searches in the most congested cliques. In addition, the eligibility conditions allow to select good channel modifications through algebraic manipulations without requiring a verification by solving the congestion control sub-problem.

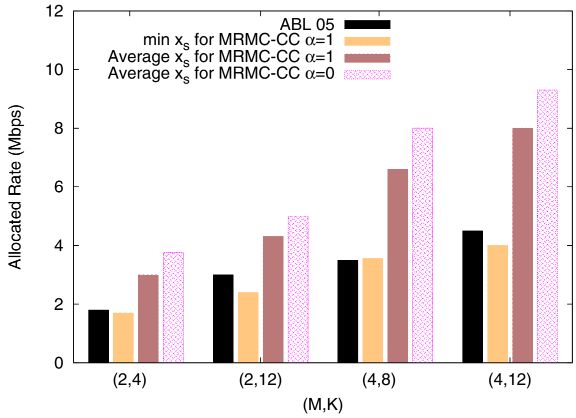
Wireless model. Our simulator supports a simple distance-based wireless channel model that maps transmission ranges to link rates and a simple interference model parameterized by maximum interference range. Although more complex wireless models are necessary, our choice is both due to lower complexity for execution in large networks and also to compare to [2] which has been evaluated using similar models.

B. Performance Evaluation

We refer to the approach in [2] as ABL05 and our approach as MRMC-CC. Given a set of source-destination pairs, both approaches compute source rates x_s and channel assignments in a multi-radio, multi-channel wireless mesh network. ABL05 also computes *node-to-node* paths between the source-destination pairs. This approach is based on a linear programming formulation and yields a maximum feasible source rate λ^* subject to the following fairness constraint: all sources are allocated a rate proportional to their weight demand vector

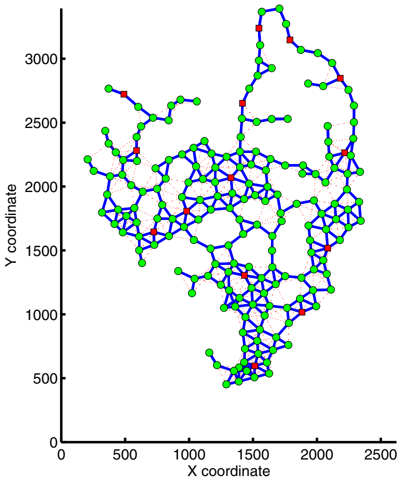


(a) Grid topology.

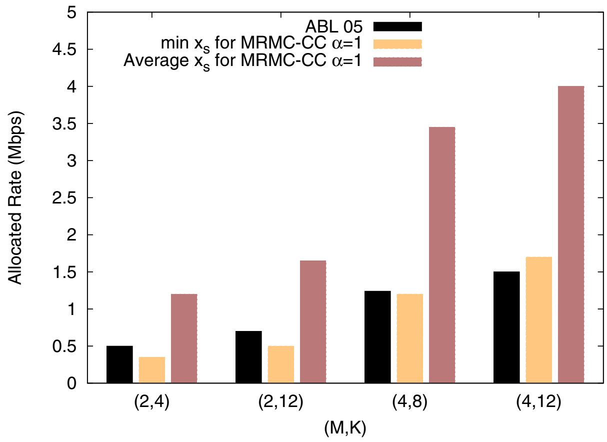


(b) Grid performance evaluation.

Fig. 4. Grid scenario topology and performance evaluation.



(a) Chaska topology.



(b) Chaska performance evaluation.

Fig. 5. Chaska scenario topology and performance evaluation.

$x_s = \lambda^* w_s$. On the other hand, our approach uses the utility-based formulation which captures a wide range of fairness objectives through both α and w_s. It also uses a single node-to-node path per source-destination pair and computes the source rates across multiple radio-to-radio paths sharing the node-to-node path. Multiple node-to-node paths can be incorporated in our utility-based formulation at the expense of increased complexity. In this paper, we pre-compute node-to-node paths separately using Dijkstra's shortest path algorithm and minimum hop count as metric.

We compare the two approaches using equal weights w_s for all sources. The main reason behind this choice is that in practice it is not straightforward to determine optimal weights in advance. In addition, ABL05 has been evaluated under this condition in [2]. The optimal (common) source rate computed by ABL05 is compared to the average and minimum rates computed by our approach for proportional fairness ($\alpha = 1$).

We use a grid topology scenario that provides uniform interference patterns and a large non-uniform topology scenario of a mesh network deployed in Chaska, MN. Below, we describe both scenarios in detail.

Grid scenario. This scenario is depicted in Fig. 4(a) and uses the same simulation setup as [2]. Nodes have equal number of 802.11a radios M, with distance-rate relationships commonly advertised by 802.11a vendors: the rates range from 6Mbps to 54Mbps with maximum transmission ranges 90m and 30m, respectively. Maximum interference range is 180m. The distance between two adjacent points in the 8x8 grid is 58.5m and 60 nodes are placed at grid points randomly. We consider various number of radios per node M and number of channels K. For each (M, K) configuration we vary the number of gateways from 2 to 12. For each gateway configuration 20 non-gateway nodes are selected randomly with equal traffic demand (20 Mbps each) toward the gateways. The reported average and minimum source rates are averages across all gateway configurations.

Chaska scenario. Fig. 5(a) depicts the 194-node topology of the city-wide 802.11b mesh network deployed in Chaska, MN. We assume the transmission range is 250m and the interference range 400m. This is a relatively large topology with non-uniform density. For computational efficiency we only use a single bit rate of 11Mbps (the maximum supported by 802.11b). In this scenario, 24 out of the 194 nodes are selected as gateways. We consider 10 different traffic matrices where all other nodes have equal and backlogged demands (11Mbps) toward the gateways. The reported average and minimum source rates are averages across all traffic matrices.

Results. Fig. 4(b) and Fig. 5(b) depict the resulting rates for various (M, K) configurations in the Grid and Chaska scenarios, respectively.

We observe that ABL05 severely under-utilizes the mesh network in both scenarios: the ABL05 rate is lower than the average MRMC-CC rate and comparable to the minimum MRMC-CC rate. The under-utilization is mainly due to two reasons. First, the fairness constraint forces equal source rates even in non-regular topologies when $w_s = 1$. Second, compared to clique-based constraints used in MRMC-CC, the link-based constraints used in ABL05 are conservative because they implicitly assume that all interfering links of each link are also interfering with respect to each other. In the grid scenario the under-utilization of ABL05 is likely more due to the link-based constraints rather than the fairness constraints because the regularity of the grid topology favors the choice $w_s = 1$ for ABL05 (equal rates). In the Chaska scenario the under-utilization of ABL05 becomes more severe as both factors come into play. It is also interesting to observe that, in both scenarios, the multiple node-to-node paths (which exploit spatial reuse) used in ABL05 cannot offset the under-utilization due to the fairness constraints and link-based contraints.

On the other hand, MRMC-CC consistently yields higher average rate than ABL05 and the difference increases as more resources (radios M and channels K) are added to the network. For the Grid scenario we have also run MRMC-CC for the objective of network-wide rate-sum optimization ($\alpha = 0$), which is expected to yield higher average rate. According to Fig. 4(b), the average rate of MRMC-CC for $\alpha = 1$ is comparable to the average rate for $\alpha = 0$. Hence, in the Grid scenario proportional fairness utilizes the network resources in an efficient manner.

VI. Conclusions

In this paper we addressed the problem of congestion control and channel assignment in multi-radio, multi-channel wireless mesh networks. We introduced a formulation that distributes traffic to a set of paths that characterize the radio and channel transmission capabilities of the network and a decomposition approach where congestion control and channel assignment sub-problems are iteratively solved to address the joint problem. We showed that this iterative approach outperforms existing approaches and yields rates and channel assignments that achieve high network utilization and span a wide range of fairness objectives.

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